



HALDIA INSTITUTE OF TECHNOLOGY

(AN AUTONOMOUS INSTITUTION UNDER MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL)

Paper Code: PCC-CS 401

Paper Name: Discrete Mathematics

Time Allotted: 2 Hours

Full Marks: 70

The figures in the margin indicate full marks

Candidates are required to give their answers in their own words as far as practicable

Group – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives from the followings:

30 x 1 = 30

(i) A relation R is defined on the pair of integers as follows: $(x, y) R (u, v)$ if and only if $x < u$ and $v > y$, Then R is, **(CO1)**

a) Reflexive b) Symmetric c) Antisymmetric d) Transitive

(ii) If G be a cyclic group containing 10 elements with generator x , then $x^{10} = ?$ **(CO2)**

a) x b) x^{-1} (Inverse of x) c) e (Identity element) d) x^9 **(CO4)**

(iii) The number of unit elements of the ring $(Z, +, \cdot)$ **(CO4)**

a) 2 b) 3 c) 1 d) Infinite

(iv) Which of the following statement is true **(CO1)**

a) Every non-empty subset of positive integer contains finite number of elements.

b) For any 2 non-empty sets A and B with $f: A \rightarrow B$ and $g: B \rightarrow A$ be injective mapping, then there exist a bijective mapping $h: A \rightarrow B$.

c) A relation is said to be equivalence relation, if it is reflexive, antisymmetric and transitive.

d) None of the above.

(v) The chromatic number of a bipartite graph having n vertices **(CO6)**

a) n b) $n - 1$ c) 2 d) Depends upon n

(vi) Let $p(x): "x \in N \text{ such that } x^2 + 2 < 12"$. The truth set of $p(x)$ is **(CO5)**

- a) $\{x: x \in N, x > 5\}$ b) $\{x: x \in N, x \leq 5\}$
 c) $\{x: x \in N, x > 3\}$ d) $\{x: x \in N, x \leq 3\}$

(vii) Find the number m of seven-letter words that can be formed using the letters of the word **BENZENE** (CO4)

- a) 420 b) 240 c) 402 d) 204

(viii) In how many ways 1024 be written as product of 3 positive integers (CO4)

- a) 22 b) 55 c) 66 d) 99

(ix) $p \wedge (q \wedge \sim p) \equiv$ (CO3)

- a) Contingency b) Contradiction c) Tautology d) None of these

(x) A connected planar graph with same number of vertices and edges determines (CO5)

- a) 1 region b) 2 regions c) 3 regions d) 4 regions

(xi) Find the minimum number of students in a class to be sure that three of them are born in the same month. (CO4)

- a) 25 b) 26 c) 36 d) 35

(xii) If a graph G has chromatic number 15, then minimum number of colors required for coloring the graph, is (CO6)

- a) 1 b) 14 c) 15 d) 16

(xiii) The number of non-zero zero divisor in $M_{2 \times 2}(R)$, with respect to matrix addition and multiplication is (CO5)

- a) 0 b) 1 c) 4 d) Infinite

(xiv) Statement-I: $n(U) = 600$, $n(A) = 450$, $n(B) = 250$ and $n(A \cap B) = 50$, then given data's are correct. (CO2)

Statement II: $n(A \cup B) \leq n(U)$.

- a) both Statement-I and Statement-II are true and Statement-II is the correct explanation of Statement-I.
 b) both Statement-I and Statement-II are true but Statement-II is not the correct explanation of Statement-I.
 c) Statement-I is true but Statement-II is false.
 d) Statement -I is false because of Statement -II is true.

(xv) The value of the expression $x + x'$ (CO3)

- a) 0 b) 1 c) x d) x'

(xvi) $\sim(p \wedge q) \equiv$ (CO3)

- a) $\sim p \wedge \sim q$ b) $p \vee qc) \sim p \vee \sim q$ d) $p \wedge q$

(xvii) A box contains 8 blue socks and 6 red socks. Find the number of ways two socks can be drawn from the box if, they can be any color. (CO4)

- a) 91 b) 48 c) 28 d) 16

(xviii) If a binary tree has 20 pendant vertices, then the number of internal vertices of the tree is **(CO6)**

- a) 20 b) 21 c) 23 d) 19

(xix) If S and T are two subgroups of a group G then $S \cup T$ is a sub-group of G if **(CO6)**

- a) $S \cup T = \emptyset$ b) $S \cap T = \emptyset$ c) $S \subset T$ d) None of these

(xx) Which of the following is not true **(CO4)**

- a) Every subgroup of an abelian group is abelian.
b) For a normal subgroup, every left coset is a right coset.
c) Intersection of two subgroups is again a subgroup.
d) Every subgroup of a cyclic group is not a cyclic.

(xxi) The number of generators of a cyclic group of order 8 is **(CO4)**

- a) 2 b) 6 c) 7 d) 4

(xxii) The necessary and sufficient conditions for a non-empty subset S of a ring R to be subring of R are **(CO4)**

- a) $a, b \in S \Rightarrow a + b \in S, ab \in S$ b) $a, b \in S \Rightarrow a - b \in S$
c) $a, b \in S \Rightarrow a \cdot b \in S$ d) $a, b \in S \Rightarrow a - b \in S, ab \in S$

(xxiii) A connected planar graph has 5 edges and 3 vertices. The number of regions determined by the graph is **(CO6)**

- a) 1 b) 2 c) 4 d) 8

(xxiv) The symmetric group S_3 has how many elements? **(CO4)**

- a) 6 b) 8 c) 9 d) None of these

(xxv) $p \vee p \equiv p$ is **(CO3)**

- a) Idempotent law b) Commutative law c) De' Morgan's law d) None of these

(xxvi) Which of the following set is closed under numerical multiplication **(CO4)**

- a) $\{1, -1, 0, 2\}$ b) $\{1, \omega, \omega^2\}$ c) $\{1, i\}$ d) $\{1, \omega\}$

(xxvii) If R is a ring without zero divisors, then $x \cdot y = 0$ implies **(CO4)**

- a) $x = 0$ or $y = 0$ b) $x = 0$ and $y = 0$ c) only $x = 0, y \neq 0$ d) only $y = 0, x \neq 0$

(xxviii) From the premises $\sim p$ and $p \vee q$ one can conclude **(CO3)**

- a) q b) p c) $\sim p$ d) $\sim q$

(xxix) Kuratowski's graph is a **(CO6)**

- a) Planar graph b) Regular Graph c) Tree d) Binary Tree

(xxx) The value of $(132) \circ (13)$, in S_3 , permutation of any 3 elements, is (CO6)

- a) $(1\ 3)$ b) $(1\ 2)$ c) $(2\ 3)$ d) $(1)(2)(3)$

Group – B
(Multiple Choice Type Questions)

2. Choose the correct alternatives from the followings:

10 x 2 = 20

(i) For a graph G , having chromatic polynomial $x(x-1)^3$, Then G is (CO6)

- a) Tree with 4 vertices c) Tree with 3 vertices
b) Tree with 5 vertices d) Complete graph with 4 vertices

(ii) Inverse of the permutation $\begin{pmatrix} 1 & 2 & 34 \\ 3 & 1 & 42 \end{pmatrix}$ is (CO4)

- a) $\begin{pmatrix} 1 & 2 & 34 \\ 1 & 2 & 34 \end{pmatrix}$ b) $\begin{pmatrix} 1 & 2 & 34 \\ 4 & 3 & 21 \end{pmatrix}$ c) $\begin{pmatrix} 1 & 2 & 34 \\ 2 & 4 & 13 \end{pmatrix}$ d) $\begin{pmatrix} 1 & 2 & 34 \\ 2 & 3 & 14 \end{pmatrix}$

(iii) Let $f: R \rightarrow R$ and $g: R \rightarrow R$ be defined as $f(x) = x^2 + 3$, $g(x) = x + 6$ then $f \circ g(x) =$

- a) $4x^2 - 6$ b) $x^2 + 12x + 39$ c) $x^2 - 6$ d) $4x^2 - 6x + 1$ (CO2)

(iv) Find the number of non-negative integral solutions of $x + y + z = 18, x \geq 3, y \geq 2, z \geq 1$ (CO2)

- a) 88 b) 89 c) 90 d) 91

(v) The chromatic polynomial of the complete graph, K_3 (CO6)

- a) $x(x-2)$ b) $x(x-1)^2$ c) $x^2(x-1)^2$ d) $x(x-1)(x-2)$

(vi) The mapping $f: R \rightarrow R$, defined by $f(x) = \frac{x}{x-a}, x \neq a$ is (CO2)

- a) one – one, $a \neq 0$ b) onto, $a \neq 0$ c) one – one, $a = 0$ d) Bijective.

(vii) The truth table of $(p \vee q) \rightarrow (p \wedge q)$ contains (CO3)

- a) 2 T 2 F b) All F c) 3 T, 1 F d) All T

(viii) $\sim(\sim p \vee \sim(q \wedge p)) \equiv$ (CO3)

- a) $\sim p \vee q$ b) $\sim p \vee \sim q$ c) $p \vee q$ d) $p \wedge q$

(ix) A car company manufactures 5 models of cars. Find the number of ways a customer can buy 8 cars. (CO4)

- a) 500 b) 495 c) 395 d) None of these

(x) Let G be a group and $a \in G$. If $O(G) = 17$ then $O(a^8)$ is (CO4)

- a) 8 b) 15 c) 17 d) 9

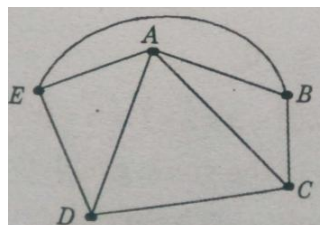
Group – C
(Short Answer Type Questions)

3. Attempt any four from the followings:

4 x 5 = 20

i) Find the chromatic polynomial of the graph: G:

(CO6)



ii) Find the minimum number of integers to be selected from $S = \{1, 2, 3, \dots, 9\}$, so that

- (a) The sum of two integers is even. **(CO2)**
- (b) The difference of two among is 5.

iii) Show that the following argument is a fallacy: $p \rightarrow q, \neg p \vdash \neg q$. **(CO3)**

iv) Show that every finite integral domain is a field. **(CO4)**

v) Define planar graph. Find the dual graph of complete graph K_4 . **(CO6)**

vi) Using Euclidean Algorithm, find the solution of Diophantine Equation: $243x + 315y = 9$.

(CO2)